

석사과정

1. (5 점) Find, up to isomorphism, all groups of order ≤ 7 .
No proof is necessary.

2. (5 점) Let A, B be $n \times n$ matrices. Prove :
(1) $\text{Tr}(AB) = \text{Tr}(BA)$, where Tr means the trace.
(2) If A is similar to B , then $\text{Tr}(A) = \text{Tr}(B)$.

3. (5 점) Prove that no group of order 30 is simple.

4. (5 점) Let R be a commutative ring with unity and $\mathfrak{p} \subsetneq R$ be its ideal. Prove that \mathfrak{p} is a prime ideal if and only if R/\mathfrak{p} is an integral domain.

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5. (6 점) Let $\Omega = \{(x_1, x_2, x_3) : x_1^2 + x_2^2 < 1, 0 < x_3 < x_1 + 2\}$, and define $F(x_1, x_2, x_3) = (x_1^3, x_2^3, x_3)$. Evaluate

$$\int_{\partial\Omega} F \cdot \vec{n} dS,$$

where \vec{n} is the outward unit normal vector to $\partial\Omega$.

6. (6 점) Find the solution $y(t)$ of the initial value problem

$$y'' + 4y' + 4y = e^t; \quad y(0) = 1, y'(0) = 3$$

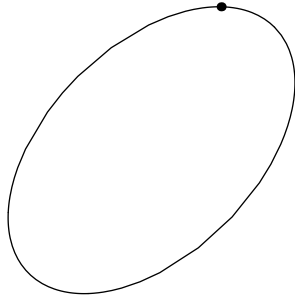
7. (6 점) Compute

$$\oint_{|z|=1} \frac{e^z}{z} dz.$$

8. (7 점) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin nx$ converges uniformly or not, and justify your answer.

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9. (6 점) 다음은 타원의 한 점 p 를 표시한 것이다.



이 타원은 원 S 에 선형변환 T 를 시행하여 얻은 것이고 (즉, 타원은 원 S 의 T -상(image)이고), 점 p 는 S 에 내접하는 정삼각형 \triangle 의 한 꼭지점의 T -상이다. 이때 \triangle 의 나머지 두 꼭지점의 T -상을 도시하라.



10. (6 점) 곡선

$$x = \cos t, \quad y = \sin t, \quad z = t$$

에서 $t = 1$ 일 때의 곡률은 얼마인가?



11. (7 점) Show that \mathbb{R}/\sim , where $x \sim y$ if $x - y$ is rational, is not Hausdorff.
pf)



12. (6 점) Show that a continuous bijective map from a compact space into a Hausdorff space is a homeomorphism.



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13. (10 점) Consider the ring $R = \{ a + b\sqrt{-1} : a, b \in \mathbb{Z} \}$, where \mathbb{Z} is the ring of integers. Answer with brief justification.
- (1) What is the characteristic of R ?
 - (2) Find all units of R .
 - (3) Is R a principal ideal domain?
 - (4) Find all prime numbers $p \in \mathbb{Z}$ which are not irreducible elements in R .



14. (10 점) Let G be a group and N be its normal subgroup. Prove that G/N is abelian if and only if N contains the commutator subgroup G' of G .



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15. (10 점) Given a function $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$$F(x_1, x_2, x_3) = (e^{x_2} \cos x_1, e^{x_2} \sin x_1, 5 - \cos x_3),$$

find all the points $P(x_1, x_2, x_3)$ where the Inverse Function Theorem holds.

16. (10 점) Compute

$$\frac{1}{2\pi i} \oint_{|z|=10} \frac{zP'(z)}{P(z)} dz,$$

where $P(z) = (z - 1)(z - 2)(z - 3)(z - 4)$.

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17. (15 점) Let M be a closed (isometrically) embedded surface in \mathbf{R}^3 . Assume that M is diffeomorphic to a torus.

- (1) Let K denote the Gaussian curvature of M . What is the $\int_M K dA$?
- (2) Is it possible that K is zero at every point of M ? Justify your answer.



18. (15 점) Prove the followings.

- (1) Prove that a metric space is normal.
- (2) Let X be a normal space and C be a closed subset of X . Show that for any neighborhood U of C , there exists a continuous function $f : X \rightarrow [0, 1]$ such that $f(C) = 1$ and $f(U^c) = 0$, where U^c is the complement of U .
- (3) In the above problem, if X is also second countable, then show that f can be chosen further that $f^{-1}(1) = C$.

