A survey of alternating permutations

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Abstract

A permutation $a_1 a_2 \cdots a_n$ of $1, 2, \ldots, n$ is *alternating* if $a_1 > a_2 < a_3 > a_4 < a_5 > \cdots$. If E_n is the number of alternating permutations of $1, 2, \ldots, n$, then

$$\sum_{n \geq 0} E_n \frac{x^n}{n!} = \sec x + \tan x.$$

We will discuss several aspects of the theory of alternating permutations. Some occurences of the numbers E_n, such as counting orbits of group actions and volumes of polytopes, will be surveyed. The behavior of the length of the longest alternating subsequence of a random permutation will be analyzed, in analogy to the length of the longest increasing subsequence. We will also explain how various classes of alternating permutations, such as those that are also fixed-point free involutions, can by counted using a certain representation of the symmetric group S_n whose dimension is E_n