Abstract: Let Z be a subordinate Brownian motion in $\mathbb{R}^d$, $d \geq 3$, via a subordinator with Laplace exponent ϕ. We kill the process Z upon exiting a bounded open set $D \subset \mathbb{R}^d$ to obtain the killed process Z^D, and then we subordinate the process Z^D by a subordinator with Laplace exponent ψ. The resulting process is denoted by Y^D. Both ϕ and ψ are assumed to satisfy certain weak scaling conditions at infinity.

In this talk, I will present some recent results on the potential theory, in particular the boundary theory, of Y^D. First, in case that D is a κ-fat bounded open set, we show that the Harnack inequality holds. If, in addition, D satisfies the local exterior volume condition, then we prove the Carleson estimate. In case D is a smooth open set and the lower weak scaling index of ψ is strictly larger than $1/2$, we establish the boundary Harnack principle with explicit decay rate near the boundary of D. On the other hand, when $\psi(\lambda) = \lambda^\gamma$ with $\gamma \in (0, 1/2]$, we show that the boundary Harnack principle near the boundary of D fails for any bounded $C^{1,1}$ open set D. Our results give the first example where the Carleson estimate holds true, but the boundary Harnack principle does not.

We also prove a boundary Harnack principle for non-negative functions harmonic in a smooth open set E strictly contained in D, showing that the behavior of Y^D in the interior of D is determined by the composition $\psi \circ \phi$.

This talk is based a joint paper with Panki Kim and Zoran Vondracek.