

QUASIMODE, EIGENFUNCTION AND SPECTRAL PROJECTION BOUNDS FOR SCHRÖDINGER OPERATORS ON MANIFOLDS WITH CRITICALLY SINGULAR POTENTIALS

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We provide quasimode estimate for $-\Delta_g + V$ on compact Riemannian manifolds (M, g) of dimension $d \geq 2$:

$$(1) \quad \|u\|_{L^p(M)} \leq C_{p,V} (\lambda^{\sigma(p)-1} \|(-\Delta_g + V - (\lambda + i)^2)u\|_{L^2(M)} + \lambda^{\sigma(p)} \|u\|_{L^2(M)}).$$

Here V is assumed to be the potential in $L^{\frac{d}{2}}$ or \mathcal{K} (the Kato class). When $V \in L^{\frac{d}{2}}$, a partial result is obtained, (1) holds for $2 < p < \frac{2d}{d-3}$. The other case, when $V \in \mathcal{K}$, we can expect more positive results, since the operator $H_V := -\Delta + V$ has a self-adjoint extension. Indeed, we have a variant of (1) for all dimensions $d \geq 2$ and all exponents $2 < p \leq \infty$ when $V \in \mathcal{K}$. The main ingredients of the proof are Hadamard parametrix approach and oscillatory integral bounds of Dos Santos Ferreira, Kenig and Salo [2]. Such estimate (1) has various applications; it could be used to obtain the spectral projection bounds, or $L^p \rightarrow L^p$ spectral multiplier bounds, or Strichartz estimates for solutions of $(\partial_t - \Delta + V)u = 0$. We will also discuss these topics if time allows.

REFERENCES

- [1] M.D. Blair, Y. Sire, C.D. Sogge, *Quasimode, eigenfunction and spectral projection bounds for schrödinger operators on manifolds with critically singular potentials*, preprint, 2019.
- [2] David Dos Santos Ferreira, Carlos E. Kenig, and Mikko Salo, *On L^p resolvent estimates for Laplace-Beltrami operators on compact manifolds*, Forum Math. **26** (2014), no. 3, 815-849.