WEIGHTED ESTIMATES FOR MAXIMAL PRODUCT OF SPHERICAL AVERAGES.

KALACHAND SHUIN

The spherical averages of a continuous function f is defined by

$$A_t f(x) := \int_{\mathbb{S}^{n-1}} f(x-ty)d\sigma(y), \text{ for } t>0.$$

Spherical averages often make their appearance as a solution of partial differential equations. For instance the average $u(x,t) = \frac{1}{4\pi}\int_{\mathbb{S}^2} tf(x-ty)\, d\sigma(y)$ is a solution of the wave equation

$$\triangle_x u(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t),$$
$$u(x,0)=0, \ \frac{\partial u}{\partial t}(x,0) = f(x),$$

in $\mathbb{R}^3$. In 1976, E.M. Stein proved that the spherical maximal operator $M_{\text{full}}f = \sup_{t>0}|A_t f|$ is bounded on $L^p(\mathbb{R}^n)$ if and only if $p > \frac{n}{n-1}$ for $n \geq 3$. Later, Bourgain extended the above boundedness result to dimension $n=2$. In the spirit of bilinear Hardy-Littlewood maximal function (considered by A. Lerner et al. 2009), we have considered the following operator

$$\mathcal{M}_{\text{full}}(f_1,f_2)(x) := \sup_{t>0}|A_t f_1(x) A_t f_2(x)|.$$

We have also considered the lacunary maximal function as follows

$$\mathcal{M}_{\text{lac}}(f_1,f_2)(x) := \sup_{j\in\mathbb{Z}}|A_{2^j} f_1(x) A_{2^j} f_2(x)|.$$

In this talk, I shall discuss sparse domination and weighted estimates of the above mentioned maximal product of spherical averages with respect to genuine bilinear weights.